

The Dynamic Aspects of Thermo-Elasto-Viscoplastic  
Snap-Through and Creep Buckling Phenomena

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by

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The problem of dynamic buckling of shallow arches and spherical caps has received considerable attention in the past thirty years. Several studies have been conducted by various investigators on impulsively loaded configurations and configurations which are suddenly loaded with loads of constant magnitude and infinite duration, but primarily of linearly elastic material behavior.

Solutions to elastic response of such structures started appearing in the open literature in the early 1950's. Hoff and Bruce [1] considered the dynamic stability of a pinned half-sine arch under a half-sine distributed load. Budiansky and Roth [2] in studying the axisymmetric behavior of a shallow spherical cap under suddenly applied loads defined the load to be critical, when the transient response increases suddenly with very little increase in the magnitude of the load. This concept was adopted by numerous investigators [3] in the subsequent years because it is tractable to computer solutions. Conceptually, one of the best efforts in the area of dynamic buckling, under impulsive and suddenly applied constant loads, is the work of Hsu and his collaborators [4,5]. In his studies, he defined sufficiency conditions for stability and sufficiency conditions for instability, thus finding upper and lower bounds for the critical impulse of critical sudden load. Independently, Simitses [6] in dealing with the dynamic buckling of shallow arches and spherical caps termed the lower bound as a minimum possible critical load and the upper bound as a minimum guaranteed critical load. The reader is also referred to Ref. [7-9] for a more comprehensive review and other effects.

The effects of inelastic material behavior found their way into the literature since the 1960's. Creep buckling of shallow arches has been investigated by Huang and Nachbar [10], and Krajcinovic [11]. The paper by Miyazaki and Ando [12] deals with creep buckling of perfect spherical shells subjected to pressure loading and considers only the effects of steady-state creep.

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Structural Dynamics

Xirochakis and Jones [13] have studied axisymmetric and bifurcation creep buckling of externally pressurized spherical shells under the condition of secondary creep only. Botros and Bienek [14] in a recent paper presented a numerical treatment of the creep buckling of these configurations. Their work includes the effects of elastic strain, primary and secondary creep strains and creep recovery. The book by Owen and Hinton [15] gives a list of references for the applications of finite element methods to the problem of creep buckling of structures.

As far as the authors know no work has been reported on the dynamic non-isothermal elasto-viscoplastic behavior of shallow arches and spherical shells.

The prediction of inelastic behavior of metallic materials at elevated temperatures has increased in importance in recent years. The operating conditions within the hot section of a rocket motor or a modern gas turbine engine present an extremely harsh thermo-mechanical environment. Large thermal and mechanical transients are induced each time the engine is started or shut down. Additional thermal transients, from an elevated ambient, occur whenever the engine power level is adjusted to meet flight requirements. The structural elements employed to construct such hot sections, as well as any engine components located therein, must be capable of withstanding such extreme conditions. Failure of a component would, due to the critical nature of the hot section, lead to an immediate and catastrophic loss in power and thus cannot be tolerated. Consequently, assuring satisfactory long term performance for such components is a major concern for the designer.

Non-isothermal dynamic loading of structures often causes excursion of stress well into the inelastic range. Moreover, the influence of geometry changes on the response is also significant in most of the cases. Therefore both material and geometric nonlinear effects must be considered.

In previous papers [16,17], the authors have presented a constitutive law for thermo-elasto-viscoplastic behavior of metallic materials, in which the main features are: (a) unconstrained strain and deformation kinematics, (b) selection of reference space and configuration for the stress tensor, bearing in mind the rheologies of real materials, (c) an intrinsic relation which satisfies material objectivity, (d) thermodynamic consistency, and (e) proper choice of external and internal thermodynamic variables.

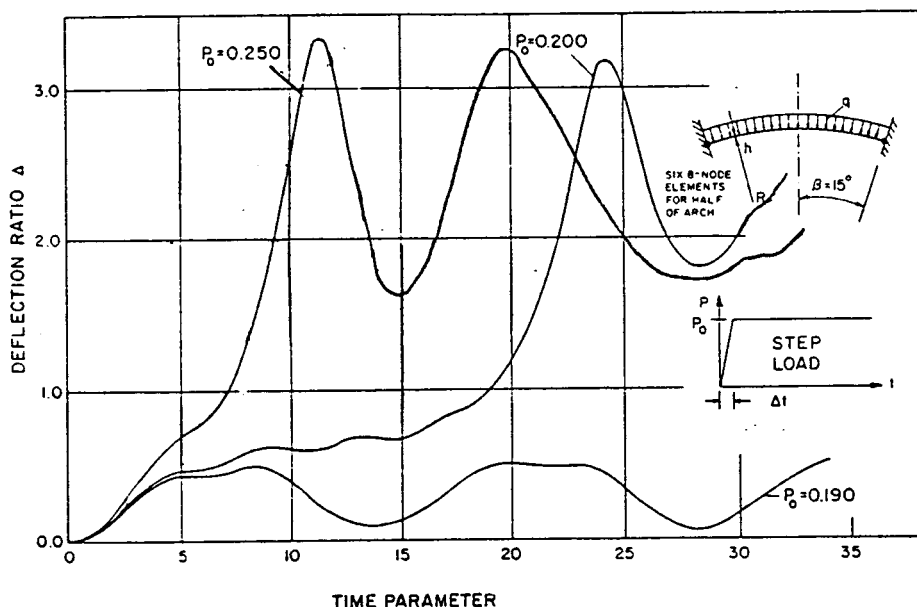
The problem of buckling of shallow arches under static thermo-mechanical loads was investigated in Ref. [17]. It was shown in [17] that the material constitutive equations are capable of capturing all non-isothermal, elasto-viscoplastic characteristics. Furthermore, the method is capable of predicting response which includes pre and postbuckling behavior due creep and plastic effects.

The present paper focuses on a mathematical model and solution methodology, to examine dynamic buckling and dynamic postbuckling behavior of shallow arches and spherical caps made of a realistic material and undergoing non-isothermal, elasto-viscoplastic deformation. Thus, geometric as well as material

type nonlinearities of higher order are included in the analysis. The dynamic stability problem is studied under impulsive loading and suddenly applied loading with loads of constant magnitude and infinite duration. The two types of loads may be thought of as mathematical idealization of blast loads (a) large decay rates and small decay times and (b) small decay rates and large decay times respectively.

A finite element model has been derived directly from the incrementally formulated nonlinear shell equations, by using a tensor-oriented procedure. The time history of the deformation is analyzed with the aid of several schemes of integration in time of the ordinary differential equation of the discretized structure, consisting of the equations of motion (global) and the constitutive equations (local). One step implicit and iterative implicit schemes have been tested.

As an example of the results, the time history of the midspan displacement of a damped shallow circular arch is presented here in. The uniformly distributed pressure is applied as a step load. The dynamic buckling of the arch occurs at that level at which a sudden increase in the deflection ratio, is observed (see Fig 1),



**Fig. 1. Dynamic Snap-Through of A Shallow Circular Arch**

The following aspects of the dynamic elasto-viscoplastic buckling of shallow arches and spherical caps have been emphasized in this work: (a) A general form of the material constitutive equation capable of reproducing all the non-isothermal elasto-viscoplastic characteristics has been employed. (b) A kinematic and finite element formulation of the problem in the rate form, valid for large displacements, rotations and strains has been developed and (c) Simple and efficient methods of integration in time of the discretized equation of motion of the structure, capable of continuation of the solution beyond unstable critical points have been accomplished and demonstrated.

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